have large errors, are in complete agreement with the physical-region plots obtained by other authors[10] as well as our own [Fig. 7(a)].

The results given here support the general validity of the Chew-Low technique. They illustrate the strong statistical requirements to which a meaningful application of this technique is subjected; they also show that pole effects seen in the physical region are confirmed by extrapolation, as required. The reverse, however, is not necessarily true: pole contributions obtained correctly through extrapolations can be washed out in the physical region by contributions other than those of poles. Extrapolation results are therefore to be used in parallel with physical-region plots. Physical-region plots, which require less statistics than do extrapolations in order to be meaningful, are more useful therefore in detecting the location, width, etc. of the eventual resonance. Extrapolation results then give additional weight to these conclusions and provide the ultimate proof that the production process was indeed a peripheral one.

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# Broken Symmetries*

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Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

## I. INTRODUCTION

In the past few years several authors have developed an idea which might offer hope of understanding the broken symmetrics that seem to be characteristic of elementary particle physics. Perhaps the fundamental Lagrangian is invariant under all symmetries, but the vacuum state[1] is not. It would then be impossible to prove the usual sort of symmetry relations among $S$-matrix elements, but enough symmetry might remain (perhaps at high energy) to be interesting.

But whenever this idea has been applied to specific models, there has appeared an intractable difficulty. For example, Nambu suggested that the Lagrangian might be invariant under a continuous chirality transformation $\psi \rightarrow \exp(i\boldsymbol{\theta}\cdot\boldsymbol{\tau}\gamma_5)\psi$ even if the fermion physical mass $M$ were nonzero. But then there would be a conserved current $J_\lambda$, with matrix element

$$\langle p'|J_\lambda|p\rangle = f(q^2)\bar{u}'\gamma_5[i\gamma_\lambda - (2M/q^2)q_\lambda]u,$$

where $q=p-p'$. The pole at $q^2=0$ can only arise from a spinless particle of mass zero, which almost certainly does not exist. Of course, the pole would not occur if $f(0)=0$, which might be the case if we do not insist on identifying $J_\lambda$ with the axial vector current of $\beta$ decay. But Nambu showed that this unwanted massless "pion" also appears as a solution of the approximate Bethe-Salpeter equation.[1]

Goldstone[2] has examined another model, in which the manifestation of "broken" symmetry was the nonzero vacuum expectation value of a boson field. (This was suggested as an explanation of the $\Delta I=\frac{1}{2}$ rule by Salam and Ward.)[3] Here again there appeared a spinless particle of zero mass. Goldstone was led to conjecture that this will always happen whenever a continuous symmetry group leaves the Lagrangian but not the vacuum invariant.

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We will present here three proofs of this result. The first uses perturbation theory; the other two are much more general.

## II. PERTURBATION THEORY

We will consider a multiplet of $n$ spinless fields $\phi_i$ which interact among themselves and perhaps also with other fields. The Lagrangian is assumed to be invariant under a set of infinitesimal transformations:

$$\delta^\alpha \phi_i = \epsilon T_{ij}{}^\alpha \phi_j. \tag{1}$$

If the vacuum state were also invariant under these transformations, the vacuum expectation values of the $\phi_i$ would be subject to a set of linear relations,

$$T_{ij}{}^\alpha \langle \phi_j \rangle_0 = 0. \tag{2}$$

(Usually, there would be enough such relations to imply that all $\phi_i$ have zero vacuum expectation value. This is the case in the example to be discussed at the end of this section, where the $\phi_i$ transform as the defining representation of the orthogonal group, so that the $T$ span the space of all antisymmetric matrices.)

We are going to examine the possibility that the vacuum state is not invariant under these transformations; in particular we will consider the consequences that ensue if

$$T_{ij}{}^\alpha \langle \phi_j \rangle_0 \neq 0 \tag{3}$$

for some $\alpha$ and some $i$. It is inconvenient to work with fields with nonzero vacuum expectation value, we we will define

$$\phi_i = \chi_i + \eta_i, \tag{4}$$

where

$$\eta_i \equiv \langle \phi_i \rangle_0,$$

so that $\chi_i$ is a quantum field with

$$\langle \chi_i \rangle_0 \equiv 0. \tag{5}$$

In perturbation theory this means that we should ignore all "tadpole" diagrams with a single external $\chi$ line.

The Lagrangian $L(\phi)$, although invariant under (1), will, in general, not be invariant under the "naive" transformations

$$\delta^\alpha \chi_i = \epsilon T_{ij}{}^\alpha \chi_j. \tag{6}$$

Hence, the vanishing of $\langle \chi_i \rangle_0$ provides a nontrivial self-consistency condition which allows us to calculate $\eta_i$ up to some unavoidable ambiguities. We will now show that the value of $\eta$ is such that propagators of some of the $\chi_i$ have a pole at zero mass.

We begin by defining a function $F(\eta)$ as the sum of all proper connected graphs with no external lines, and with the over-all energy momentum conservation factor $i(2\pi)^4\delta^4(0)$ omitted. Every factor $\lambda_i$ in each term of $F(\lambda)$ represents a place where we might instead have an external line of type $i$. This can be seen in general by noting that the interaction Lagrangian density used in calculating these graphs is

$$L'(\chi,\eta) = L(\chi+\eta) - L_0(\chi),$$
$$L_0(\chi) = -\tfrac{1}{2}(\partial_\mu \chi_i)(\partial^\mu \chi_i) - \tfrac{1}{2} m^2 \chi_i \chi_i. \tag{7}$$

It follows then that the sum $F^{(N)}$ of all connected proper diagrams with $N$ external lines $i, j, \cdots$ carrying zero energy and momentum is (for $N \neq 2$)

$$F_{ij\cdots}{}^{(N)} = (\partial^N / \partial\eta_i \partial\eta_j \cdots) F(\eta). \tag{8}$$

For $N=2$ the mass term in $-L_0$ gives an additional contribution, so

$$F_{ij}{}^{(2)} = (\partial^2/\partial\eta_i\partial\eta_j) F(\eta) + m^2 \delta_{ij}. \tag{9}$$

As defined here, $F^{(N)}$ does not include the propagators for its external lines or the over-all factor $i(2\pi)^4\delta(0)$.

It is clear from the definition of $F^{(1)}$ that

$$F_i{}^{(1)} = (\Delta'^{-1}(0))_{ij} \langle \chi_j \rangle_0. \tag{10}$$

Here $\Delta'(p)$ is the complete propagator given by

$$\Delta'_{ij}(p) = \int d^4x\, e^{-ip\cdot x} \langle T\{\chi_i(x), \chi_j(0)\} \rangle_0. \tag{11}$$

Because $\langle \chi_i \rangle_0$ vanishes, the most general improper diagram for $\Delta'(p)$ can be constructed by stringing together proper self-energy parts $\Pi^*(p)$ into a linear chain; hence the inverse of the propagator is given as usual by

$$(\Delta'^{-1}(p))_{ij} = (p^2+m^2)\delta_{ij} - \Pi^*_{ij}(p). \tag{12}$$

For zero momentum,

$$\Pi^*_{ij}(0) = F_{ij}{}^{(2)}, \tag{13}$$

and so, using (9), we have

$$(\Delta'^{-1})_{ij} = -(\partial^2/\partial\eta_i\partial\eta_j) F(\eta). \tag{14}$$

We are going to prove that (14) has no inverse, so it should be kept in mind that it is $\Delta'^{-1}(0)$ and not $\Delta'(0)$ that is well defined.

To complete the proof, we now must make use of the invariance of $L(\phi)$ under the transformation (1). This has the consequence that it is only the presence of the $\eta$ terms that breaks invariance under (6), and hence that $F(\eta)$ is invariant under the corresponding transformations

$$\delta^\alpha \eta_i = \epsilon T_{ij}{}^\alpha \eta_j. \tag{15}$$

Thus

$$(\partial F/\partial\eta_i) T_{ij}{}^\alpha \eta_j = 0. \tag{16}$$

Differentiating with respect to $\eta$, this gives

$$(\partial^2 F/\partial\eta_i\partial\eta_j) T_{ik}{}^\alpha \eta_k + (\partial F/\partial\eta_i) T_{ij}{}^\alpha = 0. \tag{17}$$

For physically allowed values of $\eta$ this relation together with (10) and (14), yields at zero momentum

$$(\Delta'^{-1})_{ij} T_{ik}{}^\alpha \eta_k = 0. \tag{18}$$

We see that for zero momentum the inverse of the propagator becomes singular and so some elements of the propagator become infinite. This does not prove that there is a pole at zero mass, but we certainly expect the propagator to be infinite at $P^2=0$ only if the theory involves particles of zero mass. The fields with nonvanishing matrix element between the vacuum and states of zero mass are

$$\chi^\alpha \equiv T_{ik}{}^\alpha \eta_k \chi_i. \tag{19}$$

Clearly, none of this trouble would occur if it were not for our assumption (3) that $T_{ik}{}^\alpha\eta_k \neq 0$. It is the broken symmetry, and not merely the nonzero vacuum expectation value $\eta$, that necessitates massless bosons.[4]

To see how this all works in a specific example, let us take as the Lagrangian

$$L(\phi)=-\bar\psi(\gamma\partial+M)\psi-\tfrac{1}{2}(\partial_\mu\phi_i)(\partial^\mu\phi_i)-\tfrac{1}{2}m^2\phi_i\phi_i \\ -g\bar\psi O_i\psi\phi_i-\tfrac{1}{4}\lambda(\phi_i\phi_i)^2. \tag{20}$$

Defining $\phi_i=\chi_i+\eta_i$, this becomes

$$L(\phi)=L(\chi)-m^2(\chi_i\eta_i)-\tfrac{1}{2}m^2(\eta_i\eta_i)-g\bar\psi O_i\psi\eta_i \\ -\lambda(\chi_i\chi_i)(\chi_j\eta_j)-\lambda(\chi_i\eta_i)^2-\tfrac{1}{2}\lambda(\chi_i\chi_i)(\eta_j\eta_j) \\ -\lambda(\eta_i\eta_i)(\chi_j\eta_j)-\tfrac{1}{4}\lambda(\eta_i\eta_i)^2. \tag{21}$$

The low-order contributions to the sum of all proper connected vacuum graphs are

$$F(\eta)=F(0)-\tfrac{1}{2}m^2(\eta_i\eta_i)-\tfrac{1}{4}\lambda(\eta_i\eta_i)^2+i(2\pi)^{-4}g\eta_i \\ \times\int d^4p\ \mathrm{Tr}\{O_iS(p)\}+i(2\pi)^{-4}\lambda(\eta_i\eta_i+\tfrac{1}{2}\delta_{ij}\eta_k\eta_k) \\ \times\int d^4p\ \Delta_{ij}(p)+\frac{i}{2}(2\pi)^{-4}g^2\eta_i\eta_j\int d^4p \\ \times\mathrm{Tr}\{O_iS(p)O_jS(p)\}-i(2\pi)^{-4}\lambda^2(\eta_i\eta_j+\tfrac{1}{2}\delta_{ij}\eta_k\eta_k) \\ \times(\eta_a\eta_b+\tfrac{1}{2}\delta_{ab}\eta_c\eta_c)\int d^4p\ \Delta_{ia}(p)\Delta_{jb}(p) \\ -(2\pi)^{-8}\lambda^2\delta_{ij}\eta_k\delta_{ab}\eta_c\int d^4p\ d^4p' \\ \times\{\Delta_{ia}(p)\Delta_{jb}(p')\Delta_{kc}(p+p')+\Delta_{ic}(p)\Delta_{jb}(p') \\ \times\Delta_{ka}(p+p')+\Delta_{ib}(p)\Delta_{jc}(p')\Delta_{ka}(p+p'\}, \tag{22}$$

where

$$S(p)=[-ip_\mu\gamma^\mu+M]^{-1}, \\ \Delta_{ij}(p)=\delta_{ij}(p^2+m^2)^{-1}.$$

It can be readily seen that the derivative of $F(\eta)$ with respect to $\eta_i$ is the sum $F_i{}^{(1)}$ of the proper connected diagrams with one external line, and that the second derivative with respect to $\eta_i$ and $\eta_j$ is the sum of the proper connected diagrams with two external lines, except that the $m^2\eta^2$ term in $F(\eta)$ does not contribute to $F^{(2)}$.

We will now assume that $L(\phi)$ is invariant under the group $SO(n)$ of orthogonal transformations on $\phi$. This implies that the operators $O_i$ are such that

$$\int d^4p\ \mathrm{Tr}\{O_iS(p)\}=0, \tag{23}$$

$$\int d^4p\ \mathrm{Tr}\{O_iS(p)O_jS(p)\}=\delta_{ij}I. \tag{24}$$

Thus $F(\eta)$ is a function of $\eta^2=(\eta_i\eta_i)$,

$$F(\eta^2)=F(0)-\tfrac{1}{2}m^2\eta^2-\tfrac{1}{4}\lambda\eta^4 \\ +i(2\pi)^{-4}\eta^2(1+\tfrac{1}{2}n)\int d^4p(p^2+m^2)^{-1} \\ +\tfrac{1}{2}i(2\pi)^{-4}g^2\eta^2I-i(2\pi)^{-4}\lambda^2\eta^4(2+\tfrac{1}{4}n) \\ \times\int d^4p(p^2+m^2)^{-1}(p^2+m^2)^{-1}-(2\pi)^{-8}\lambda^2\eta^2(2+n) \\ \times\int d^4p\int d^4p'(p^2+m^2)^{-1}(p'^2+m^2)^{-1} \\ \times[(p+p')^2+m^2]^{-1}. \tag{25}$$

The dependence on $\eta^2$ alone implies then that

$$F_i{}^{(1)}=2\eta_iF', \tag{26}$$

$$F_{ij}{}^{(2)}=(m^2+2F')\delta_{ij}+4\eta_i\eta_jF''. \tag{27}$$

(A prime denotes differentiation with respect to $\eta^2$.) We see that the contribution of the term $-\tfrac{1}{2}m^2\eta^2$ in $F^{(2)}$ is canceled by the term $m^2\delta_{ij}$ in (27) arising from $-L_0$.

We have shown that $F_i{}^{(1)}$ is proportional to $\langle\chi_i\rangle_0$ and so must vanish. This implies that either $\eta$ is zero or it must satisfy the consistency condition:

$$F'(\eta^2)=0. \tag{28}$$

Thus if $\eta$ is not zero, it is determined up to an orthogonal transformation; we certainly could not expect a more unambiguous determination.[5]

[4] It is clear from (19) that the maximum number of zero-mass fields is $L$, the number of Lie generators. There may in special cases be fewer than $L$ zero-mass fields if not all fields $\chi^\alpha$ given by (19) are linearly independent. This happens for example when $T_{ik}{}^\alpha$ correspond to the "tensor" representations of simple Lie groups. For this case $T_{ik}{}^\alpha$ are antisymmetric for all three indices. Therefore $\eta_\alpha\chi^\alpha=\eta_\alpha T_{ik}{}^\alpha\eta_k\chi_i=0$, and only $(L-1)$ of the fields $\chi^\alpha$ are linearly independent. These results are unaltered even if we allow in the theory more than one set of scalar fields $\phi_i$ with nonzero vacuum expectation values. To take a concrete case, the spurion theory proposed by Salam and Ward (reference 3) to explain the $\Delta I=\tfrac{1}{2}$ rule rests on assuming $\langle K_1{}^0\rangle\neq0$. This would mean that the three companion fields to $K_1{}^0$, i.e., $K^-$, $K^+$, and $K_2{}^0$, must possess zero masses.

[5] Basic to the entire self-consistency procedure is, of course, the conjecture that $F'(\eta^2)=0$ does possess a root for real $\eta$. By considering classical field theories, Goldstone states that a real root would exist provided the bare mass for the $\phi_i$ fields is pure imaginary. It is interesting to note that if the Lagrangian (20) contains no $-\tfrac{1}{4}\lambda(\phi_i\phi_i)^2$ term, an application of Lehmann's mass theorem [H. Lehmann, Nuovo cimento **11**, 342 (1945)] shows that Goldstone's condition [(bare mass)$^2<0$] can never be satisfied. If, however, the $-\tfrac{1}{4}\lambda(\phi_i\phi_i)^2$ term is present in the Lagrangian, Lehmann's theorem gives no indication of the sign of (bare mass)$^2$.

We have also shown that

$$\begin{aligned}\Delta_{ij}'^{-1}(0)&=m^2\delta_{ij}-F_{ij}^{(2)}\\&=-2F'\delta_{ij}-4\eta_i\eta_jF''.\end{aligned}\tag{29}$$

If $\eta=0$, there is no reason to expect that $F'=0$, so that $(\Delta'^{-1})_{ij}$ is the nonsingular matrix $-2F'\delta_{ij}$. But if $\eta\neq0$, then for physically allowed values of $\eta$ we must have $F'=0$ so,

$$(\Delta'^{-1}(0))_{ij}=-4\eta_i\eta_jF''.\tag{30}$$

This is certainly a singular matrix. In fact

$$(\Delta'^{-1}(0))_{ij}u_j=0\tag{31}$$

for any $u$ orthogonal to $\eta$. All such $u$ can be expressed as

$$u_i=T_{ij}\eta_j,\tag{32}$$

by choosing $T_{ij}$ as an appropriate antisymmetric matrix. We see then that the space of $u$'s is precisely the space indicated by the general considerations above.

### III. GENERAL PROOFS

If the Lagrangian is invariant under an $n$-dimensional set of infinitesimal transformations which transform a general field $\phi_a$ according to

$$\delta\phi_a=\epsilon T_{ab}{}^{\alpha}\phi_b,\tag{33}$$

then there will exist a set of conserved currents

$$J^{\mu\alpha}=i\frac{\partial L}{\partial(\partial_\mu\phi_a)}T_{ab}{}^{\alpha}\phi_b,\tag{34}$$

$$\partial_\mu J^{\mu\alpha}=0.\tag{35}$$

The usual proof of the conservation equations (35) makes use only of the invariance of the Lagrangian, and hence should not be affected by the noninvariance of the vacuum. Also, from the canonical commutation relations we always expect that

$$[Q^\alpha,\phi_a]=T_{ab}{}^{\alpha}\phi_b,\tag{36}$$

where

$$Q^\alpha=\int d^3x\,J^{0\alpha}(x).\tag{37}$$

We will begin by assuming again that there exists a set of spinless fields $\phi_i$ transforming according to Eq. (1), i.e.,

$$[Q^\alpha,\phi_i]=T_{ij}{}^{\alpha}\phi_j.\tag{38}$$

The fields $\phi_i$ need not be "fundamental" here; all our remarks will apply equally well if the $\phi_i$ are synthetic objects like $\bar{\psi}O_i\psi$.

We shall show that if the vacuum is not annihilated by $Q^\alpha$, so that

$$T_{ij}{}^{\alpha}\langle\phi_j\rangle\neq0,\tag{39}$$

then the theory must involve massless particles.

The place we will look for zero-mass singularities is in the vacuum expectation value of the commutator of $J^{\mu\alpha}$ with $\phi_i$. Using the usual Lehmann-Källén arguments, this can be written

$$\langle[J^{\mu\alpha}(x),\phi_i(y)]\rangle_0=\partial^\mu\int dm^2\,\Delta(x-y,m^2)\rho_i{}^{\alpha}(m^2),\tag{40}$$

where $\Delta$ is the usual causal Green's function for mass $m$ and

$$(\Box^2-m^2)\Delta(x-y,m^2)=0,\tag{41}$$

$$\begin{aligned}(2\pi)^{-3}p^\mu\theta(p^0)\rho_i{}^{\alpha}(-p^2)=-\sum\delta(p-p^n)(0|J^{\mu\alpha}{}_{(0)}|n)\\\times\langle n|\phi_i(0)|0\rangle.\end{aligned}\tag{42}$$

The current conservation condition (35) together with (40) and (41) implies that

$$m^2\rho_i{}^{\alpha}(m^2)=0,\tag{43}$$

and hence

$$\rho_i{}^{\alpha}(m^2)=N_i{}^{\alpha}\delta(m^2),\tag{44}$$

$$\langle[J^{\mu\alpha}(x),\phi_i(y)]\rangle_0=N_i{}^{\alpha}\partial^\mu D(x-y),\tag{45}$$

where $D$ is $\Delta$ for $m=0$. We would normally expect no singularity in $\rho(m^2)$, or in other words, $N_i{}^{\alpha}=0$. (It is well known, for example, that the pion-decay matrix element would vanish if the axial vector current were conserved.) But because of (39) we can show that $N_i{}^{\alpha}\neq0$. For

$$\begin{aligned}N_i{}^{\alpha}&=\int d^3x\,N_i{}^{\alpha}\partial^0D(x)\\&=\langle[Q^\alpha,\phi_i(0)]\rangle_0\\&=T_{ij}{}^{\alpha}\langle\phi_j(0)\rangle_0\neq0.\end{aligned}\tag{46}$$

Thus the sum in (42) must include states of zero mass.

It perhaps does not necessarily follow from the noninvariance of the vacuum that there exists (or can be constructed) a set of spinless fields $\phi_i$ with $T_{ij}{}^{\alpha}\langle\phi_j\rangle_0\neq0$. We therefore wish to offer a simple nonrigorous argument that if there were no massless particles in the theory, then we would have to conclude that

$$0=|\alpha\rangle\equiv Q^\alpha|0\rangle;\tag{47}$$

for the conservation of current implies that $Q^\alpha$ and hence $|\alpha\rangle$ is invariant under the inhomogeneous Lorentz group. But then

$$\langle\alpha|J^{\alpha\mu}(x)|0\rangle=0,\tag{48}$$

so that

$$\langle\alpha|\alpha\rangle=\int d^3x\,\langle\alpha|J^{\alpha0}(x)|0\rangle=0.\tag{49}$$

Equation (47) follows from (49) and the positive-definiteness assumption.

This "proof" is probably unobjectionable in ordinary theories with no massless particles. But if there are massless particles, the integrals in (37) and (49) become somewhat poorly defined, because then there are states that are not Lorentz invariant but arbitrarily close to Lorentz invariance. If $|\alpha\rangle$ is such a state, the matrix element (48) will be small, but may give a large value to the integral (49).

As an example, let $|\mathbf{p}\rangle$ be a state containing a particle of mass zero, and construct the wave packet

$$|f\rangle=\int d^3p\ |\mathbf{p}|^{-\frac{1}{2}}f(|\mathbf{p}|)|\mathbf{p}\rangle. \tag{50}$$

The normalization condition is

$$1=4\pi\int|f(E)|^2EdE, \tag{51}$$

so that if we wish we can choose $f(0)\neq 0$. Lorentz invariance requires that

$$\langle\mathbf{p}|J^{\mu\alpha}(x)|0\rangle=N^\alpha|\mathbf{p}|^{-\frac{1}{2}}p^\mu e^{-ip\cdot x}, \tag{52}$$

where $p^0=|\mathbf{p}|$.

For particles with mass, current conservation would require that $N^\alpha=0$, but no such conclusion can be drawn for massless particles. For the wave packet (52), we now have

$$\langle f|J^{\mu\alpha}(x)|0\rangle=N^\alpha\int d^3p(p^\mu/|p|)f^*(|\mathbf{p}|)e^{-ip\cdot x},$$

so that

$$\langle f|\alpha\rangle=\int d^3x\,\langle f|J^{\mu\alpha}(x)|0\rangle$$
$$=(2\pi)^3N^\alpha f^*(0).$$

If we choose $f(0)$ to be nonzero, then the factor $|\mathbf{p}|^{-\frac{1}{2}}$ in (50) gives $|f\rangle$ a more-or-less Lorentz-invariant component, which has nonzero matrix element with $|\alpha\rangle$.

This becomes a bit more understandable if we ask ourselves what is the meaning of the state

$$(\exp i\epsilon Q^\alpha)\ |0\rangle=|0\rangle+i\epsilon\ |\alpha\rangle.$$

Clearly, this state is degenerate with $|0\rangle$, and is in fact another possible vacuum. It involves an infinitesimal component containing massless bosons of preponderantly low momentum. The role of the massless particles is apparently just to give meaning to the various possible vacua.

## IV. PROSPECTS FOR THE UNSYMMETRIC VACUUM

The general proofs of the last section rest entirely on the assumption that there exists a conserved current, and that the integral of its time-component satisfies (38). This follows formally from the invariance of the Lagrangian, but in a *quantum* field theory the noncommutativity of the factors in the current, and the possible nonconvergence of the integral of its time-component, make our arguments essentially nonrigorous.

Therefore, it seems reasonable to defer belief in the necessity of massless bosons in a theory with unsymmetric vacua until such a *bête noire* is found in an actual calculation based on such a theory. We have already shown in Sec. II that the massless bosons do appear when we perform calculations using perturbation theory, provided that the symmetry of the theory is broken only by the choice of the vacuum expectation value $\eta$ of the boson field.

But this is not the most general possibility. The original work of Nambu[1] indicates that the choice of a fermion mass can also break a symmetry. In this theory the fermion mass is

$$-ip^\mu\gamma_\mu=m_1+i\gamma_5m_2,$$

where $(m_1,m_2)$ transform under chirality transformations like the components of a 2-vector. If $m_1$ and $m_2$ are not zero they must satisfy a condition of form

$$F(m_1^2+m_2^2)=0.$$

Any particular choice of direction for the vector $(m_1,m_2)$ breaks the chirality invariance. (It should be noted that Nambu's choice $m_2=0$ is purely arbitrary and not dictated by parity conservation. For a general mass we must simply define the matrix associated with parity transformations to be

$$[(m_1+im_2\gamma_5)/(m_1^2+m_2^2)^{\frac{1}{2}}]\beta,$$

rather then just $\beta$.)

In Nambu's theory there is no "bare" spinless boson, but it is possible to construct a two-vector

$$\phi_1=\bar\psi\psi,\quad \phi_2=i\bar\psi\gamma_5\psi.$$

With Nambu's definition of parity (i.e., $m_2=0$) the vacuum expectation value of $\phi_2$ but not of $\phi_1$ vanishes, so the vector $\langle\boldsymbol{\phi}\rangle_0$ points in the 1-direction. An infinitesimal chirality transformation would rotate $\langle\boldsymbol{\phi}\rangle_0$ towards the 2-axis, so we are led to conjecture that the propagator of $\phi_2$ has a zero-mass pole. In fact, just such a pole was found by Nambu in an approximate treatment of the bound-state problem. However, to show that the pole remains at zero mass when more complicated diagrams are considered would require a more thorough understanding of the treatment of bound states in perturbation theory. We are attempting this at present.

In a more complicated situation we could have an invariance broken both by the choice of a vacuum expectation value of a "bare" field and also simultaneously by the choice of a mass. For example, if we specialize the model discussed in Sec. II to the case of chirality invariance, we must take

$$M=0,\quad O_1=1,\quad O_2=i\gamma_5,$$

so that

$$L=-\bar\psi(\partial\cdot\gamma)\psi-\tfrac{1}{2}(\partial_\mu\phi_i)(\partial^\mu\phi_i)-\tfrac{1}{2}m^2(\phi_i\phi_i)$$
$$-g\bar\psi\psi\phi_1-ig\bar\psi\gamma_5\psi\phi_2-\tfrac{1}{4}\lambda(\phi_i\phi_i)^2.$$

In this case our conjecture would be that:

(1) If part of the loss of symmetry is due to the choice of a two-vector $\langle\phi\rangle_0$, then there must appear a zero-mass pole in the part of $\phi$ perpendicular to this vacuum expectation value.

(2) If part of the loss of symmetry is due to the choice of a nonzero Fermion mass $m_1+im_2\gamma_5$ then there must appear a zero-mass pole in the propagator of

$$\phi'=-m_2\bar{\psi}\psi+m_1\bar{\psi}\gamma_5\psi.$$

[Presumably this is the same pole as for (1). Parity conservation would require $(m_1,m_2)$ to be in the direction of $\langle\phi\rangle_0$.]

(3) If part of the loss of symmetry is due to the choice of a noninvariant boson mass (i.e., if the residue of the pole at mass $m$ in the propagator of $\phi_i$ and $\phi_j$ is a matrix which is not just a constant times $\delta_{ij}$), then there must appear a *two-boson* pole at zero mass in the propagator of $\phi^2$.

These "conjectures" can be taken as proved if we accept the arguments of Sec. III. We believe that we will also soon be able to prove these conjectures, in general, within the framework of perturbation theory.

If this is so, then there seem only three roads open to an understanding of broken symmetries based on the noninvariance of the vacuum:

(A) The particle interpretation of such theories might be revised (as in the Gupta-Bleuler method) so that the massless particles are not physically present in final states if they are absent in initial states. However, all our attempts in this direction have failed.

(B) The massless particles might really exist. The argument against this based on the Eötvös experiment might not apply if the particles carry quantum numbers, since then the scattering cross section of two macroscopic bodies due to exchange of the massless bosons would be proportional only to the numbers of atoms in each body and not (as for Coulomb forces or gravitation) to the squares of the numbers of atoms. But the couplings of these massless particles would presumably be quite strong, and would have shown up in exotic decay modes.

(C) Goldstone has already remarked that nothing seems to go wrong if it is just discrete symmetries that fail to leave the vacuum invariant. A more appealing possibility is that the "ur symmetry" broken by the vacuum involves an inextricable combination of gauge and space-time transformations.

*Note added in proof.* Recently, one of us (S. W., Proceedings of the 1962 Geneva Conference on High Energy Nuclear Physics) has developed a method of rewriting any Lagrangian in order to introduce fields for bound as well as "elementary" particles. This allows the proof of Sec. II to be extended to the case where the field with nonvanishing vacuum expectation value is any scalar function of the elementary particle fields, hence completing our argument.

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